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# INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI

## EE5024 – Machine Learning for Image Processing

MARKS: 40 Midterm Examination TIME: 2 Hours

**There are 4 questions and answer ALL questions. Specify clearly the key intermediate steps.**

**1.** There is an image $I_{in}$ of size 100 X 100, and 8 bits per pixel are used for storing its intensity values. 10% of its total pixels have an intensity value of 0, 10% of total pixels have an intensity of 20, 20% of pixels have an intensity of 100, 30% of pixels have an intensity of 150, and the remaining 30% of pixels have an intensity of 200.

**a)** Draw the probability density function (pdf) of $I_{in}$ for varying intensity values.

**b)** Let ***r*** be an intensity value in the input image, and ***s(r)*** be the intensity value to which ***r*** got mapped to in the histogram equalized image. Perform histogram equalization on $I_{in}$, and summarize your results by filling the following table. Mention the intermediate steps in the computation of the histogram equalization.

| *r* | *s(r)* |
|---|---|
| 0 | |
| 20 | |
| 100 | |
| 150 | |
| 200 | |

**c)** Let ***r*** be an intensity value in the input image, and ***g(r)*** be the intensity value to which ***r*** got mapped to in the resulting contrast stretched image. Perform contrast stretching on $I_{in}$, and summarize your results by filling the following table. Mention the intermediate steps in the computation of contrast stretching.

| *r* | *g(r)* |
|---|---|
| 0 | |
| 20 | |
| 100 | |
| 150 | |
| 200 | |

**d)** Draw a rough sketch of pdfs of histogram equalized image in part-b, and contrast stretched image in part-c. Write your inferences from these results.

**2.** Let ***I[u, v]*** represent an input 2D image, ***k[u, v]*** be the kernel used to convolve or correlate the input image.

**a)** Let ***G[u, v]*** and ***H[u, v]*** respectively represent the images obtained as a result of convolving and correlating ***I[u, v]*** with ***k[u, v]***. Let the indices of the kernel are varying from -W to -W both in X and Y directions. Write the mathematical expressions for ***G[u, v]*** and ***H[u,v]***.

**b)** The following figure is a cropout of an input image. Specify the *correlation kernel* ***k[u, v]*** for performing Sobel horizontal edge detection. For the central pixel marked in yellow in the following image, compute the result of correlating it with ***k[u, v]***. (No need to compute the correlation output at all pixels and it is enough to compute at the central pixel.)

| | | |
|---|---|---|
| 11 | 15 | 12 |
| 17 | ***10*** | 18 |
| 14 | 16 | 13 |

**c)** For the ***k[u, v]*** and the input image in part-b, for the central pixel marked in yellow in the image, compute the result of *convolving* it with ***k[u, v]***. Is it still possible to detect a horizontal edge from this convolution result also? Justify your answer based on the convolution and correlation results.

**3.** Consider a two-class problem with a 1D feature vector $\vec{x}$. Assume that the prior probabilities of both classes are the same. Also, assume that pdf of the likelihood function of each class can be modeled by a 1-D Gaussian distribution. Let the mean and the standard deviation of the likelihood function of the first class be 0 and 1 respectively. Similarly, let the mean and the standard deviation of the likelihood function of the second class be 2 and 2 respectively.

**a)** Compute the decision boundaries (points) that result in minimum error probability for those two classes in terms of $\vec{x}$.

**b)** Let the penalty associated with falsely classifying a sample originating from class-1 to class-2 be 1. Let the penalty for falsely classifying a sample originating from class-2 to class-1 be 0.5. With these penalties, compute the decision points that result in minimizing the average risk.

**c)** Draw a rough sketch of the pdfs of likelihood functions of both the classes. Also, mark on it, the decision points obtained in part-a and part-b with approximate labeling of their values. Draw your inferences by comparing the decision points obtained in part-b and part-c.

**4.** In a three-class 2D feature vector problem, the mean vectors for each class are $[0.1,\ 0.1]^T$, $[2.1,\ 1.9]^T$, and $[-1.5,\ 2.0]^T$ respectively. Assume that all the three classes are equiprobable. Let $\vec{x} = [1.6,\ 1.5]^T$ be the feature vector to be classified into one of the three classes.

**a)** If each class is normally distributed with the same *diagonal covariance matrix*, and also with all its diagonal elements having the same value ($\sigma^2$), classify $\vec{x}$ according to the Bayes minimum error probability classifier. Mention the intermediate steps in your computations.

**b)** If each class is normally distributed with the same covariance matrix:

$$\Sigma = \begin{bmatrix} 1.2 & 0.4 \\ 0.4 & 1.8 \end{bmatrix},$$

then classify $\vec{x}$ according to the Bayes minimum error probability classifier. Mention the intermediate steps in your computations.