$$l_i \triangleq \sum_{k=1}^{M} \lambda_{ki}\, P(\vec{x}|\omega_k)\, P(\omega_k) \quad \text{--- (11)}$$

$$\vec{x} \in R_i \quad \text{if} \quad l_i < l_j \;, \; \forall j \neq i \quad \text{--- (12)}$$

Consider $M=2$

Assign $\vec{x}$ to ω_1 if $l_1 < l_2$

$$\Rightarrow \lambda_{11} P(\vec{x}|\omega_1) P(\omega_1) + \lambda_{21} P(\vec{x}|\omega_2) P(\omega_2) < \lambda_{12} P(\vec{x}|\omega_1) P(\omega_1) + \lambda_{22} P(\vec{x}|\omega_2) P(\omega_2)$$

$$\Rightarrow (\lambda_{11} - \lambda_{12})\, P(\vec{x}|\omega_1)\, P(\omega_1) < (\lambda_{22} - \lambda_{21})\, P(\vec{x}|\omega_2)\, P(\omega_2)$$

In practice, $\lambda_{ii} < \lambda_{ij}, \; \forall j \neq i$

$\therefore$ multiply with (-1) on both sides of the above inequality

$$\Rightarrow (\lambda_{12} - \lambda_{11})\, P(\vec{x}|\omega_1)\, P(\omega_1) > (\lambda_{21} - \lambda_{22})\, P(\vec{x}|\omega_2)\, P(\omega_2)$$

$$\Rightarrow \text{If} \; \boxed{\frac{P(\vec{x}|\omega_1)}{P(\vec{x}|\omega_2)} > \frac{(\lambda_{21} - \lambda_{22})\, P(\omega_2)}{(\lambda_{12} - \lambda_{11})\, P(\omega_1)}} \quad \text{--- (13)}$$

$\Rightarrow$ then assign $\vec{x}$ to ω_1

else,

assign $\vec{x}$ to ω_2

$\dfrac{P(\vec{x}|\omega_1)}{P(\vec{x}|\omega_2)}$ is referred to as likelihood ratio

Special Case -(1)

Let $\lambda_{11} = \lambda_{22} = 0$; $\lambda_{12} = \lambda_{21}$

Substituting these values in Eq-(13) $\Rightarrow$

If $\dfrac{P(\vec{x}|\omega_1)}{P(\vec{x}|\omega_2)} > \dfrac{P(\omega_2)}{P(\omega_1)}$, assign $\vec{x}$ to ω_1

$\Rightarrow$ If $P(\vec{x}|\omega_1) \cdot P(\omega_1) > P(\vec{x}|\omega_2)\, P(\omega_2)$

then, assign $\vec{x}$ to ω_1

else, assign $\vec{x}$ to ω_2

Notice that if $P(\omega_1) = P(\omega_2)$,
the above result is same as minimizing the total error probability (P_e).

The penalty values (λ_{ij}) are usually represented with a Loss matrix (L)

$$L = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1M} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ \lambda_{M1} & \lambda_{M2} & \cdots & \lambda_{MM} \end{bmatrix}_{M \times M} \quad \text{—— (14)}$$

The diagnal element of this matrix are usually zero.

For this special case,

$$L = \begin{bmatrix} 0 & \lambda \\ \lambda & 0 \end{bmatrix}$$

<u>Special Case - 2</u>

$M=2$; $\lambda_{11} = \lambda_{22} = 0$; $\lambda_{12} \neq \lambda_{21}$; $P(\omega_1) = P(\omega_2) = 1/2$

$\Rightarrow$ Assign $\vec{x}$ to ω_1 if

$$\frac{P(\vec{x}|\omega_1)}{P(\vec{x}|\omega_2)} > \left(\frac{\lambda_{21}}{\lambda_{12}}\right)$$

$$\Rightarrow \text{If } \boxed{P(\vec{x}|\omega_1) > \left(\frac{\lambda_{21}}{\lambda_{12}}\right) P(\vec{x}|\omega_2)}$$

$\Rightarrow$ assign $\vec{x}$ to ω_1

For $l = 1$, the decision line x_0 is computed from:

$$P(x_0|\omega_1) = \left(\frac{\lambda_{21}}{\lambda_{12}}\right) P(x_0|\omega_2)$$

If $\lambda_{12} > \lambda_{21}$ (e.g. $\omega_1 \in$ malignant tumor, $\omega_2 =$ benign tumor)

$\Rightarrow \dfrac{\lambda_{21}}{\lambda_{12}} < 1$.

$\Rightarrow$ When compared to the decison line obtained by minimizing P_e, it moves to the right of that line.