Bayesian Classification Part-6

Univariate Gaussian fn

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Multivariate Gaussian Distribution:

$\vec{x}$: l-D feature vector

$$p(\vec{x}) \triangleq \frac{1}{(\sqrt{2\pi})^{l} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} \underbrace{\underbrace{(\vec{x}-\vec{\mu})^T}_{1\times l} \underbrace{\underbrace{\Sigma^{-1}}_{l\times l} \underbrace{(\vec{x}-\vec{\mu})}_{l\times 1}}_{l\times 1}}_{1}\right)$$

$\Sigma \triangleq$ covariance matrix : $l \times l$

$$\underset{l\times 1}{\vec{x}} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_l \end{bmatrix} ; \quad \underset{l\times 1}{\vec{\mu}} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_l \end{bmatrix}$$

Special case: $l = 2$

$$p(\vec{x}) = p(x_1, x_2) = \frac{1}{(2\pi) |\Sigma|^{1/2}} \exp\left(\frac{-1}{2} \begin{bmatrix} x_1-\mu_1 & x_2-\mu_2 \end{bmatrix} \Sigma^{-1} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}\right)$$

$$\vec{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} E[x_1] \\ E[x_2] \end{bmatrix}$$

$$\underset{l\times l}{\Sigma} = E\left[\begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix} \begin{bmatrix} x_1-\mu_1 & x_2-\mu_2 \end{bmatrix}\right]$$

$$= E\begin{bmatrix} (x_1-\mu_1)^2 & (x_1-\mu_1)(x_2-\mu_2) \\ (x_1-\mu_1)(x_2-\mu_2) & (x_2-\mu_2)^2 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} E[(x_1-\mu_1)^2] & E[(x_1-\mu_1)(x_2-\mu_2)] \\ E[(x_1-\mu_1)(x_2-\mu_2)] & E[(x_2-\mu_2)^2] \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$$

If x_1 and x_2 are $\underbrace{\text{statistically independent}}_{p(x_1,x_2) = p(x_1)\cdot p(x_2)}$ $\Rightarrow \sigma_{12} = 0$

Isocurves:

$$\mu_1 = \mu_2 = 0 \Rightarrow [0, 0], \quad \text{Assume } \sigma_{12} = 0$$

$$p(\vec{x}) = p(x_1, x_2) = \frac{1}{(2\pi)\,|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}\begin{bmatrix} x_1-\mu_1 & x_2-\mu_2 \end{bmatrix} \Sigma^{-1} \begin{bmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{bmatrix}\right)$$

$$-\frac{1}{2}\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 \\ 0 & \frac{1}{\sigma_2^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 \\ 0 & \frac{1}{\sigma_2^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = C$$

$$\boxed{\frac{x_1^2}{\sigma_1^2} + \frac{x_2^2}{\sigma_2^2} = C} \Rightarrow \text{Ellipse}$$

$$\Sigma = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

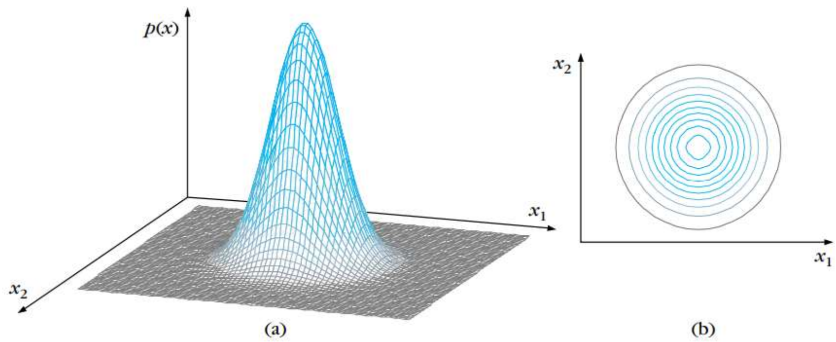

FIGURE 2.3

(a) The graph of a two-dimensional Gaussian pdf and (b) the corresponding isovalue curves for a diagonal Σ with $\sigma_1^2 = \sigma_2^2$. The graph has a spherical symmetry showing no preference in any direction.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

with $\sigma_1^2 = 15 >> \sigma_2^2 = 3$.

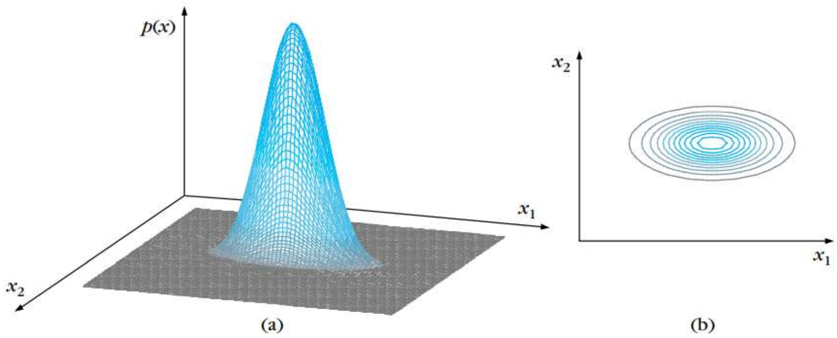

FIGURE 2.4

(a) The graph of a two-dimensional Gaussian pdf and (b) the corresponding isovalue curves for a diagonal Σ with $\sigma_1^2 >> \sigma_2^2$. The graph is elongated along the x_1 direction.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$$

$\sigma_1^2 = 3 << \sigma_2^2 = 15.$

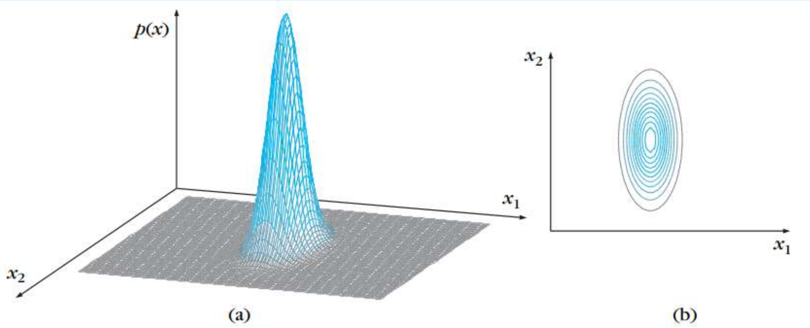

FIGURE 2.5

(a) The graph of a two-dimensional Gaussian pdf and (b) the corresponding isovalue curves for a diagonal Σ with $\sigma_1^2 << \sigma_2^2$. The graph is elongated along the x_2 direction.

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$$

$\sigma_1^2 = 15,\ \sigma_2^2 = 3,\ \sigma_{12} = 6.$

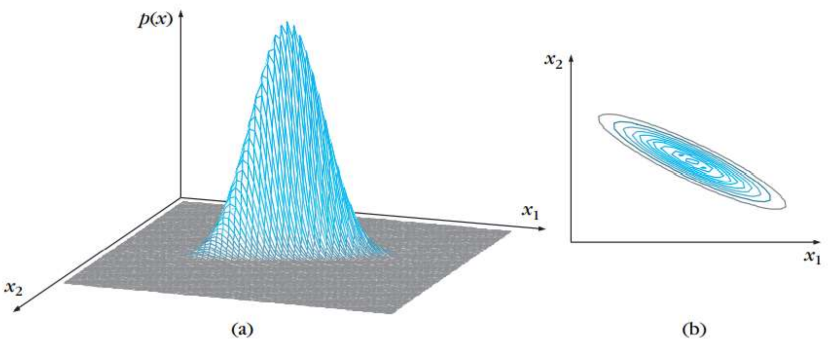

FIGURE 2.6

(a) The graph of a two-dimensional Gaussian pdf and (b) the corresponding isovalue curves for a case of a nondiagonal Σ. Playing with the values of the elements of Σ one can achieve different shapes and orientations.