Bayesian Classification Part-8

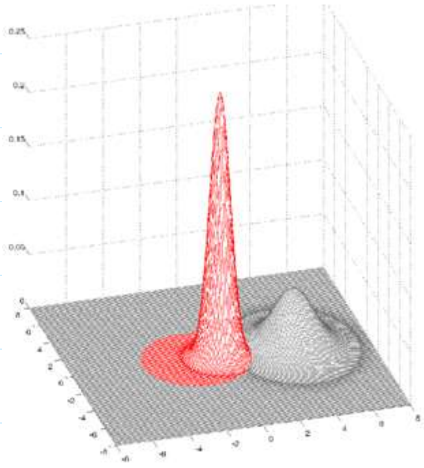
Bayesian classifier for Normally Distributed Classes:

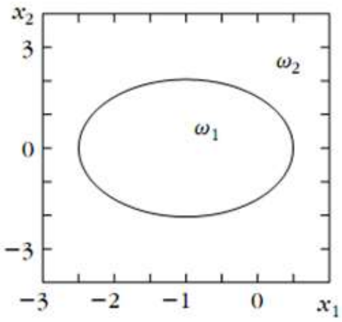
$$g_i(\vec{x}) = -\frac{1}{2\sigma_i^2}\left(x_1^2 + x_2^2 - 2\left(\mu_{i1}x_1 + \mu_{i2}x_2\right) + \left(\mu_{i1}^2 + \mu_{i2}^2\right)\right)$$
$$+ \ln(P(\omega_i)) - \frac{1}{2}\ln(|\Sigma_i|)$$

$$\vec{\mu}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \vec{\mu}_2 = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.35 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.85 \end{bmatrix}$$

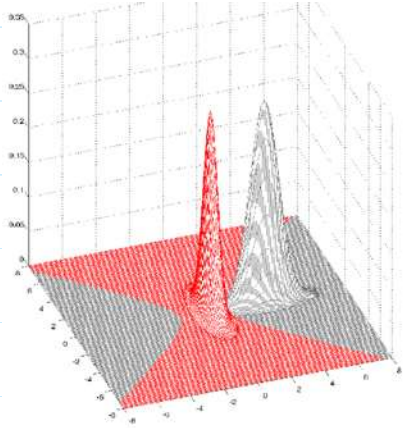


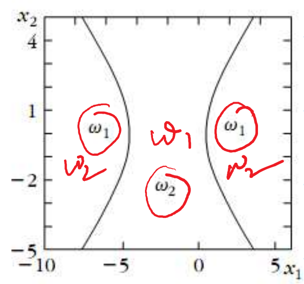


$$\vec{\mu}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \vec{\mu}_2 = \begin{bmatrix} 3.2 \\ 0 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.75 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.1 \end{bmatrix}$$





$$g_i(\vec{x}) = -\frac{1}{2}(\vec{x} - \vec{\mu}_i)^T \Sigma_i^{-1} (\vec{x} - \vec{\mu}_i) + \ln(P(\omega_i)) - \cancel{\frac{\ell}{2}\ln(2\pi)} - \frac{1}{2}\ln(|\Sigma_i|)$$

Decision Hyper Planes (Special Cases)

let us further assume that the covariance matrices are same for all classes.

$$\Sigma_i = \Sigma$$

$$\Rightarrow g_i(\vec{x}) = -\frac{1}{2\sigma^2}\left((\vec{x}^T - \vec{\mu}_i^T)(\vec{x} - \vec{\mu}_i)\right) + \ln(P(\omega_i)) - \frac{1}{2}\cancel{\ln(|\Sigma|)}$$

dropping this term

$$\Rightarrow g_i(\vec{x}) = \frac{-1}{2\sigma^2}\left(\cancel{\vec{x}^T\vec{x}} - \vec{x}^T\vec{\mu}_i - \vec{\mu}_i^T\vec{x} + \vec{\mu}_i^T\vec{\mu}_i\right) + \ln(P(\omega_i))$$

dropping this term

$$\boxed{g_{ij}(\vec{x}) = g_i(\vec{x}) - g_j(\vec{x})}$$

$$\Rightarrow g_i(\vec{x}) = \frac{1}{\sigma^2}\left(\vec{\mu}_i^T\vec{x}\right) \underbrace{- \frac{1}{2\sigma^2}\left(\vec{\mu}_i^T \cdot \vec{\mu}_i\right) + \ln(P(\omega_i))}_{w_{i0}}$$

$$\Rightarrow \boxed{g_i(\vec{x}) = W_i^T\vec{x} + w_{i0}}$$

$\Rightarrow$ Discriminant Function is <u>LINEAR</u>

$$g_{ij}(\vec{x}) = (w_i^T - w_j^T)\vec{x} + (w_{i0} - w_{i1})$$

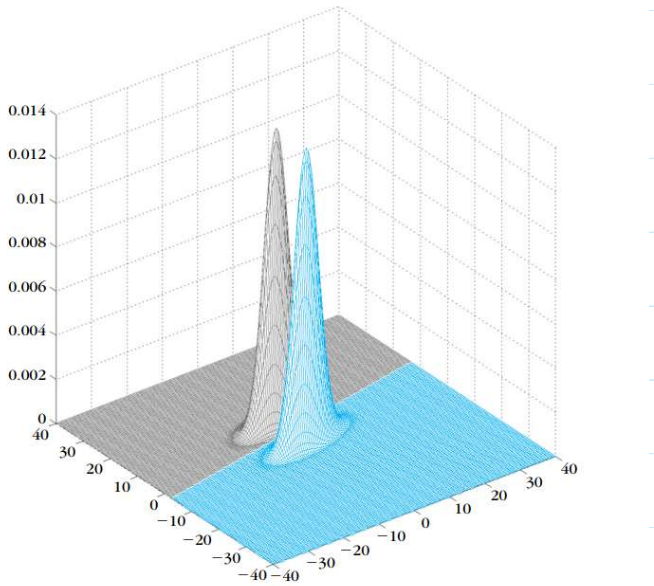


FIGURE 2.12

An example of two Gaussian pdfs with the same covariance matrix in the two-dimensional space. Each one of them is associated with one of two equiprobable classes. In this case, the decision curve is a straight line.