$$g_i(\vec{x}) = \left[\frac{1}{\sigma^2}\left(\vec{\mu}_i^T \vec{x}\right) - \frac{1}{2\sigma^2}\left(\mu_i^T \cdot \vec{\mu}_i\right) + \ln\left(p(\omega_i)\right)\right] \times \sigma^2$$

Decision Hyper Plane $(g_{ij}(\vec{x}))$

$$g_i(\vec{x}) = \vec{\mu}_i^T \vec{x} - \frac{1}{2}\left(\vec{\mu}_i^T \vec{\mu}_i\right) + \sigma^2 \ln\left(p(\omega_i)\right)$$

$$g_j(\vec{x}) = \vec{\mu}_j^T \vec{x} - \frac{1}{2}\left(\vec{\mu}_j^T \vec{\mu}_j\right) + \sigma^2 \ln\left(p(\omega_j)\right)$$

$$\Rightarrow \quad g_{ij}(\vec{x}) \triangleq g_i(\vec{x}) - g_j(\vec{x}) = 0$$

$$= \left(\vec{\mu}_i^T - \vec{\mu}_j^T\right)\vec{x} - \frac{1}{2}\left(\vec{\mu}_i^T \cdot \vec{\mu}_i - \vec{\mu}_j^T \vec{\mu}_j\right) + \sigma^2 \ln\left(\frac{p(\omega_i)}{p(\omega_j)}\right) = 0$$

$$= \left(\vec{\mu}_i - \vec{\mu}_j\right)^T \vec{x} - \frac{1}{2}\left(\vec{\mu}_i - \vec{\mu}_j\right)^T\left(\vec{\mu}_i + \vec{\mu}_j\right) + \sigma^2 \ln\left(\frac{p(\omega_i)}{p(\omega_j)}\right) = 0$$

$$\Rightarrow g_{ij}(\vec{x}) = \left(\vec{\mu}_i - \vec{\mu}_j\right)^T\left[\vec{x} - \underbrace{\left(\frac{1}{2}\left(\vec{\mu}_i + \vec{\mu}_j\right) - \sigma^2 \ln\left(\frac{p(\omega_i)}{p(\omega_j)}\right)\frac{\left(\vec{\mu}_i - \vec{\mu}_j\right)}{\left(\vec{\mu}_i - \vec{\mu}_j\right)^T\left(\vec{\mu}_i - \vec{\mu}_j\right)}\right)}_{\vec{x}_0}\right]$$

$$\Rightarrow \boxed{g_{ij}(\vec{x}) = \left(\vec{\mu}_i - \vec{\mu}_j\right)^T\left(\vec{x} - \vec{x}_0\right)}$$

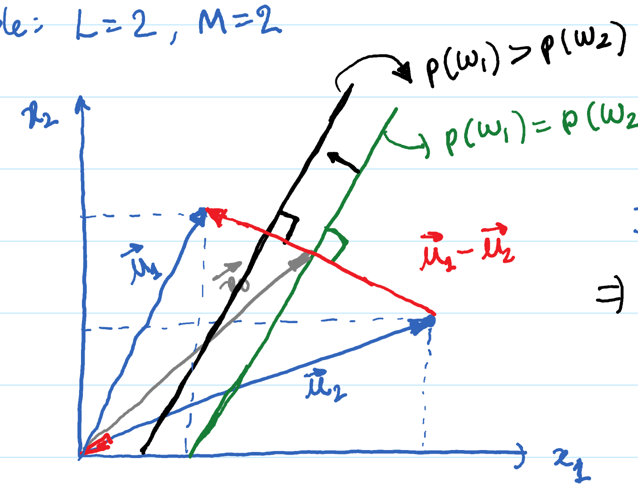
Decision Hyperplane: $g_{ij}(\vec{x}) = 0 \Rightarrow \vec{x} = \vec{x}_0$

If $p(\omega_i) = p(\omega_j)$

$\Rightarrow \quad \underline{\vec{x}_0 = \frac{1}{2}\left(\vec{\mu}_i + \vec{\mu}_j\right)}$, $\left(\vec{\mu}_i - \vec{\mu}_j\right)$ is perpendicular to $\left(\vec{x} - \vec{x}_0\right)$

$\Rightarrow$ Decision plane is perpendicular to $\left(\vec{\mu}_i - \vec{\mu}_j\right)$

Example: $L = 2$, $M = 2$



If $p(\omega_1) > p(\omega_2) = ?$

$\Rightarrow$ Decision plane moves closer to μ_1

Nondiagonal Covariance Matrix

$$g_i(\vec{x}) = \vec{\mu}_i^T \Sigma^{-1} \vec{x} - \frac{1}{2} \vec{\mu}_i^T \Sigma^{-1} \vec{\mu}_i + \ln(P(\omega_i))$$

If Σ is diagonal matrix, then $\Sigma^{-1} = \frac{1}{\sigma^2}(I)$

$$\Sigma = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} \Rightarrow \Sigma^{-1} = \frac{1}{\sigma^4} \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} = \frac{1}{\sigma^2}(I)$$

Decision plane: $g_{ij}(\vec{x}) = g_i(\vec{x}) - g_j(\vec{x}) = 0$

$$\Rightarrow \vec{\mu}_i^T \Sigma^{-1} \vec{x} - \vec{\mu}_j \Sigma^{-1} \vec{x} - \frac{1}{2} \vec{\mu}_i^T \Sigma^{-1} \vec{\mu}_i + \frac{1}{2} \vec{\mu}_j^T \Sigma^{-1} \vec{\mu}_j + \ln\left(\frac{P(\omega_i)}{P(\omega_j)}\right) = 0$$

$$\Rightarrow (\vec{\mu}_i^T - \vec{\mu}_j^T) \Sigma^{-1} \vec{x} - \frac{1}{2}(\vec{\mu}_i^T - \vec{\mu}_j^T) \Sigma^{-1} (\vec{\mu}_i + \vec{\mu}_j) + \ln\left(\frac{P(\omega_i)}{P(\omega_j)}\right) = 0$$

$$\Rightarrow \underbrace{(\vec{\mu}_i - \vec{\mu}_j)^T \Sigma^{-1}}\left(\vec{x} - \left[\frac{1}{2}(\vec{\mu}_i + \vec{\mu}_j) - \ln\left(\frac{P(\omega_i)}{P(\omega_j)}\right) \frac{(\vec{\mu}_i - \vec{\mu}_j)}{\underbrace{(\vec{\mu}_i - \vec{\mu}_j)^T \Sigma^{-1} (\vec{\mu}_i - \vec{\mu}_j)}_{\text{Mahalanobis distance}}}\right]\right) = 0$$