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Machine Learning for Image Processing

Assignment-2

1. In a two-class one-dimensional problem, the pdfs are the Gaussians $= N(0, \sigma^2)$ and $N(1, \sigma^2)$ for the two classes, respectively. Show that the threshold x_0 minimizing the average risk is equal to

$$x_0 = 1/2 - \sigma^2 \ln \frac{\lambda_{21} P(\omega_2)}{\lambda_{12} P(\omega_1)}$$

where $\lambda_{11} = \lambda_{22} = 0$ has been assumed.

2. (a) Consider a two equiprobable class, one-dimensional problem with samples distributed according to the Rayleigh pdf in each class, that is,

$$p(x|\omega_i) = \begin{cases} \frac{x}{\sigma_i^2} \, exp\left(\frac{-x^2}{2\sigma_i^2}\right) & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Compute the decision boundary point $g(x) = 0$.

(b) If $\sigma_1 = 10$ and $\sigma_2 = 2$, calculate the corresponding decision boundary and plot a rough sketch mentioning labels for both the classes along with the decision boundary.

3. Suppose that we have n independent observations $x_1, x_2, \ldots\ldots x_n$ from an exponential distribution whose density function is

$$p(x|\theta) = \begin{cases} \theta e^{-\theta x} & x \geq 0 \\ 0 & otherwise \end{cases}$$

   a. Define the likelihood of θ and write out an expression for the likelihood $L(\theta)$ of θ
   b. Find the maximum likelihood estimate $\hat{\theta}$ of θ.

4. Find the maximum likelihood estimate $\hat{\theta}$ of θ for n independent observations from Maxwell distribution whose density function is given by

$$p(x|\theta) = \begin{cases} \frac{4}{\sqrt{\pi}} \theta^{3/2} x^2 e^{-\theta x^2} & x \geq 0 \\ 0 & otherwise \end{cases}$$

5. The random variable x is normally distributed as $N(\mu, \sigma^2)$ with μ being the unknown parameter described by the Rayleigh pdf

$$p(\mu) = \frac{\mu \exp\left(-\frac{\mu^2}{2\sigma_\mu^2}\right)}{\sigma_\mu^2}$$

Show that the maximum a posteriori probability estimate of μ is given by

$$\hat{\mu}_{MAP} = \frac{Z}{2R}\left(1 + \sqrt{1 + \frac{4R}{Z^2}}\right)$$

where

$$Z = \frac{1}{\sigma^2}\sum_{k=1}^{N} x_k \,, \quad R = \frac{N}{\sigma^2} + \frac{1}{\sigma_\mu^2}$$

6. Show that, for a two-dimensional multivariate Gaussian distribution, if the feature vectors are mutually independent. It can be expressed as a multiplication of two univariate Gaussian distributions.